STA 101: Mathematics for Business and Social Sciences I

Chapter 5: Linear Programming Problem (LPP)

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STA 101 TEST 3

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VENUES:240/11

EXAMPLE 2

Objective fn → max $P = 140x + 100y$ ← profit function; x, y ← decision variable

subject to

constraints

$$c_1: \quad 5x + 2y \le 420 \quad \leftarrow \text{graphical method}$$

$$c_2: \quad 2x + 3y \le 300$$

$$x \ge 0,\ y \ge 0 \quad \leftarrow \text{non-negative constraints (always state them)}$$

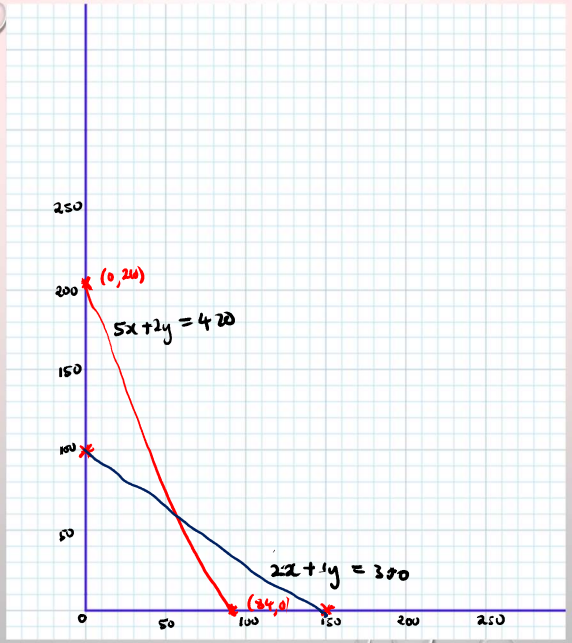
$0 + 2y = 420$, $\frac{2y}{2} = \frac{420}{2}$, $y = 210$

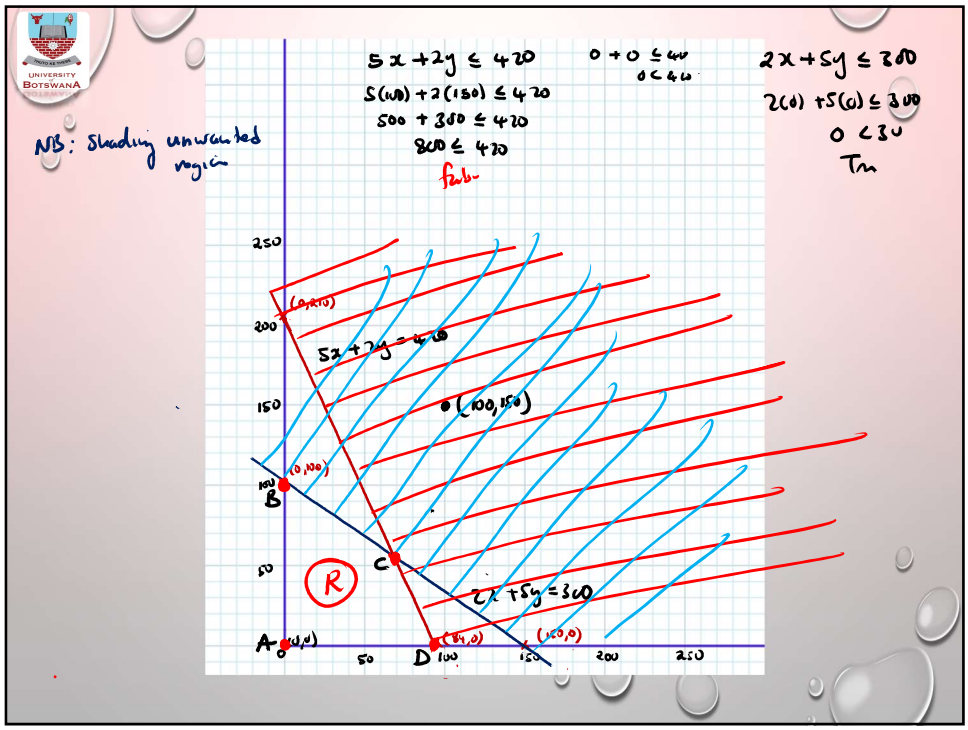
$5x + 2y = 420$

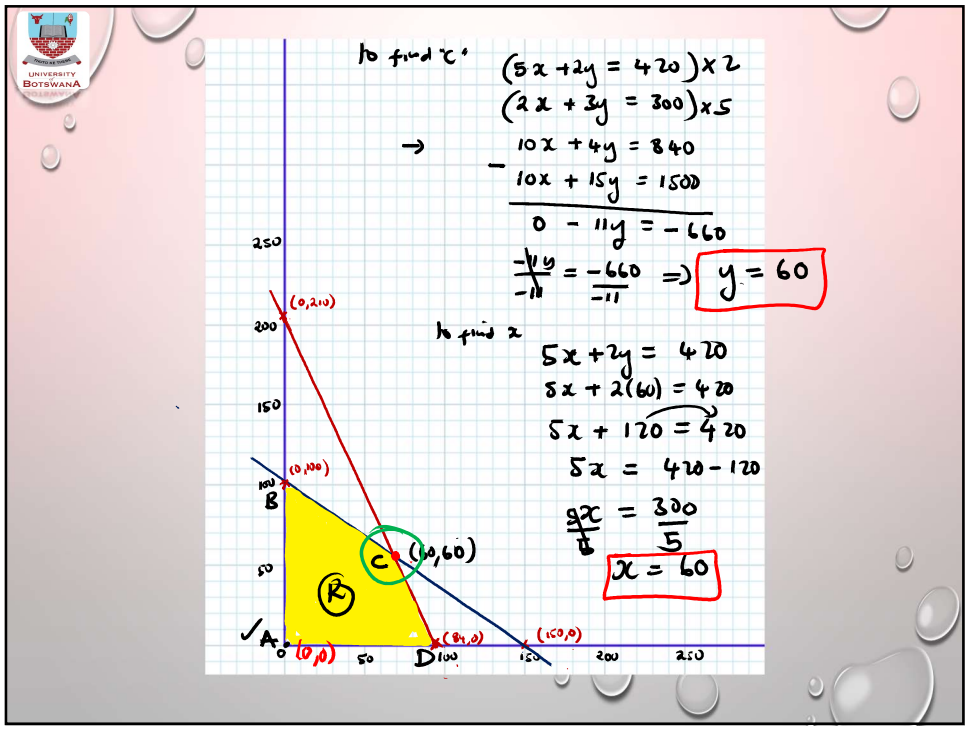
x	y	x, y
0	210	(0, 210)
84	0	(84, 0)

$2x + 3y = 300$

x	y	
0	100	(0, 100)
150	0	(150, 0)




NB: Shading unwanted region
5x + 2y ≤ 420
5(100) + 2(150) ≤ 420
500 + 300 ≤ 420
800 ≤ 420
false
0 + 0 ≤ 420
0 ≤ 420
2x + 5y ≤ 300
2(0) + 5(0) ≤ 300
0 ≤ 300
True
250
200
150
100
50
(0,210)
5x + 2y = 420
(100,150)
(0,100)
B
C
R
2x + 5y = 300
A
(0,0)
(84,0)
(150,0)
D
50
100
150
200
250


To find 'C'
(5x + 2y = 420) x 2
(2x + 3y = 300) x 5
→
10x + 4y = 840
− 10x + 15y = 1500
0 − 11y = −660
−11y/−11 = −660/−11 ⇒ y = 60
To find x
5x + 2y = 420
5x + 2(60) = 420
5x + 120 = 420
5x = 420 − 120
5x/5 = 300/5
x = 60
(0,210)
(0,100)
B
C
(60,60)
R
A
(0,0)
(84,0)
(150,0)
D
250
200
150
100
50
50
100
150
200
250

Vertex (Corners)	Points coordinate (x,y)	Objective function $P = 140x + 100y$ (max)
A	(0,0)	$P = 140(0) + 100(0) = 0$
B	(0,100)	$P = 140(0) + 100(100) = 10000$
C	(60,60)	$P = 140(60) + 100(60) = 14\,400$
D	(84,0)	$P = 140(84) + 100(0) = 11760$

The profit is maximized when $x = 60$ and $y = 60$ and profit at those points is P 14 400-00 ✗✗

DIY EXERCISE
SOLVE THE L.P.P USING THE GIVEN INFORMATION

We want to maximize the *objective function* i.e.:

thus, Maximize $P = 12x + 8y$

subject to the following constraints

$5x + 2y \leq 130$

$2x + 3y \leq 102$

$4x + 3y \leq 120$

$x \geq 0$ and $y \geq 0$

Formulation steps of a L.P.P in two variables

We go through the following steps while solving a L.P.P. in two variables by the graphical method.

- Step 1: Formulate the L.P.P using the given information
- Step 2: Represent the constraints of the L.P.P. in the (x-y) plane using a graph.
- Step 3: Identify the feasible region and the vertices of the convex region corresponding to the feasible region
- Step 4: Evaluate the value of the objective function at these vertices
- Step 5: Find the optimum (maximum or minimum) value of the objective function and the value of the corresponding decision variable at this vertex.

EXAMPLE3: (Bounded feasible region): Furniture Problem

A furniture company makes and sells tables (x) and chairs (y). Each table requires 5 board-feet of oak, 2 board-feet of pine, and 4 labour-hours. Each chair requires 2 board-feet of oak, 3 board-feet of pine and 3 labour hours. There are 130 board feet of oak, 102 board feet of pine and 120 labour hours available for the upcoming week. The profit per table is P12 while profit per chair is P8.

max $P = 12x + 8y$

i) Formulate the furniture company production problem as a linear programming problem.

ii) Draw a graph showing the feasible region.

iii) Using the graph, find the number tables and chairs should the company make during the next week so as to maximize its profit. What is the profit at this production level?

- **Solution**

- Let **x** be number of tables to be produced next week and
- **y** be the number of chairs to be produced next week

EXAMPLE3: (Bounded feasible region): Furniture Problem

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	Tables(x)	Chairs(y)	Available Resources
Oak board-feet	5	2	≤ 130
Pine board-feet	2	3	≤ 102
Labour hours	4	3	≤ 120
Profit per unit(Pula)	12	8	/// // //

Consider the table of resources

	Tables(x)	Chairs(y)	Available resources
c_1	Oak board- feet	5	2	≤ 130
c_2	Pine board -feet	2	3	≤ 102
c_3	Labour hours	4	3	≤ 120
→	Profit per unit (Pula)	12	8	

objective fn : max $P = 12x + 8y$

constraints

Oak : $5x + 2y \leq 130$

Pine : $2x + 3y \leq 102$

labour : $4x + 3y \leq 120$

$x \geq 0, y \geq 0$

We want to maximize the ***objective function*** i.e.:

thus, Maximize $P=12x + 8y$

subject to the following constraints

$5x + 2y \leq 130$

$2x + 3y \leq 102$

$4x + 3y \leq 120$

$x \geq 0$ and $y \geq 0$

ii) Graph showing the feasible region i.e. to represent the constraints of the L.P.P. on the (x-y) plane using a graph.

Plot the regions:

Equation (1) : $5x + 2y = 130$

x	y
0	65
26	0

(0,65)
(26,0)

Equation (2): $2x + 3y = 102$

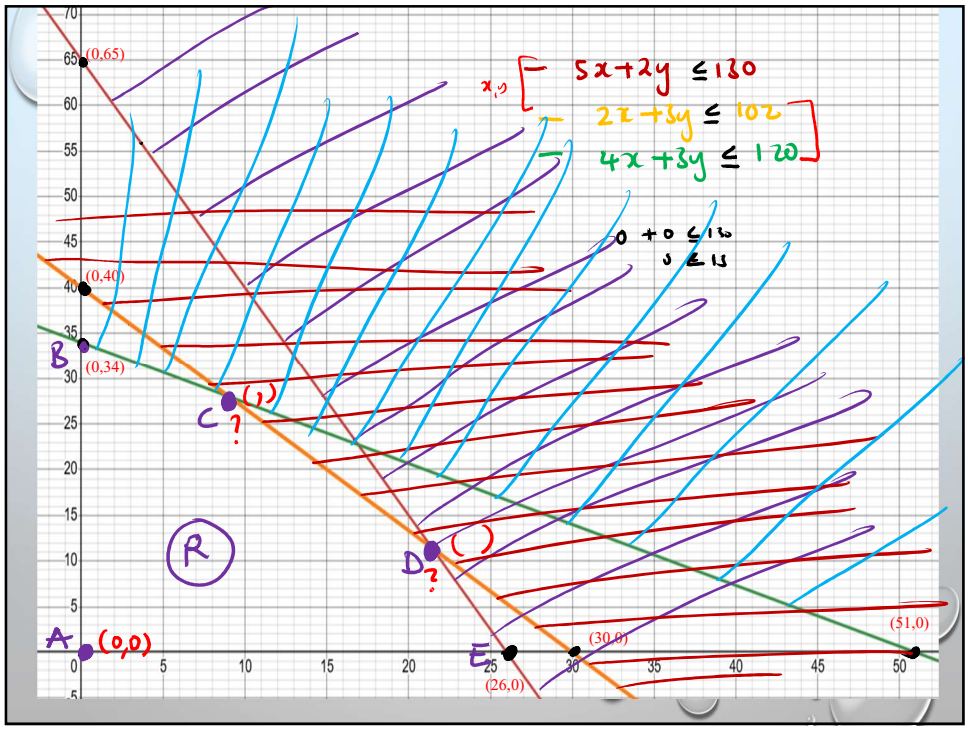
x	y
0	34
51	0

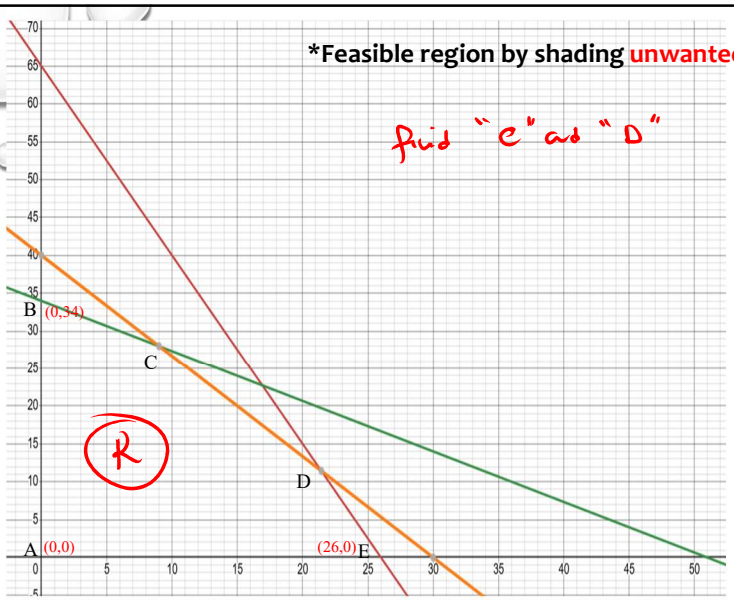
(0,34)
(51,0)

Equation (3) : $4x + 3y = 120$

x	y
0	40
30	0

(0,40)
(30,0)





The region bounded by vertices ABCDE satisfies the constraints of the given problem. Therefore, the solution to the problem lies in this bounded feasible region.

iii) Evaluate the value of the objective function at these vertices

Let us look into the vertices of this shaded region. There are 5 vertices altogether. The coordinates and the corresponding value of the objective function are given below.

Vertex (Corners)	Points coordinate (x,y)	Objective function
A		
B		
C		
D		
E		